

Name: \_\_\_\_\_

# Solutions

This assignment consists of seven questions, each worth five points for a total of 35 points. To receive full credit you must **show all necessary work**. You should write your answers in the spaces provided, but if you require more space please *staple any extra sheets* you use to this assignment. If you are having trouble with any of the problems, look at the lecture notes and exercises in the lecture notes for help. Remember to start this assignment early, your next quiz is based on this assignment.

**Algebra Review:**

1. Simplify the following expressions;

(a)  $(x+1)(x-1) + 2x$

$$= x^2 - 1 + 2x$$

Answer: \_\_\_\_\_

$$x^2 + 2x - 1$$

(b)  $(x+2)(x-1) - x(x+1)$

$$= x^2 + x - 2 - x^2 - x$$

Answer: \_\_\_\_\_

$$-2$$

(c)  $\frac{(x+3)^3 \cancel{(x-1)^2}}{\cancel{(x-1)^2} ((x+1)(x+9) - 4x)}$

$$= \frac{(x+3)^3}{x^2 + 10x + 9 - 4x} = \frac{(x+3)^3}{x^2 + 6x + 9} = \frac{(x+3)^3}{(x+3)^2}$$

Answer: \_\_\_\_\_

$$x+3$$

2. Solve the following quadratic equations, using any methods you wish. If you think there are no solutions, say "No solutions."

(a)  $x^2 + 5x + 4 = 0$

$$\Rightarrow (x+1)(x+4) = 0$$

Answer:  $x = -1, -4$

(b)  $2x^2 + x - 4 = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4(2)(-4)}}{2(2)}$$

$$= \frac{-1 \pm \sqrt{33}}{4}$$

Answer:  $x = \frac{-1 + \sqrt{33}}{4}, \frac{-1 - \sqrt{33}}{4}$

(c)  $x^2 - 2x + 3 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(3)}}{2(1)} = \frac{2 \pm \sqrt{-8}}{2} \leftarrow \text{-ve in } \sqrt{\phantom{x}}$$

Answer:  $\text{No solutions}$

**Functions:**

3. The population of a city,  $P$ , in millions is a function of  $t$ , where  $t$  is the number of years since 1980. That is,  $P = f(t)$ . Explain, in words, what the meaning of  $f(35) = 12$  is, in terms of the population of this city.

$f(35) = 12 \Rightarrow$  The population in 2015 was 12 million

4. A city's population was  $30,700$  in the year 2000 and is expected to grow at a rate of  $850$  people per year.
- (a) Give a formula for the city's population,  $P$ , as a function of  $t$ , the number of years since 2000.

Answer:  $P(t) = 850t + 30,700$

- (b) What will the predicted population of the city be in 2010?

$$P(10) = 850(10) + 30,700$$

Answer:  $39,200$

- (c) Predict the year when this city's population will reach 45,000.

$$P(t) = 45,000 \Rightarrow 850t = 14,300$$

$$\Rightarrow 850t + 30,700 = 45,000 \Rightarrow t \approx 16.82$$

Answer:  $2016$  (will accept 2017)

5. A company releases laptop in 2004. The value of the laptop decreases linearly per year. In 2008 the laptop retailed at \$1,200. In 2011 the price had fallen to \$1,025.

- (a) Based on this change in price, find a function that represents the price of the laptop as a function of  $t$  years since 2004.

2 points  $(4, 1,200)$   
 $(7, 1,025)$

$$m = \frac{1,200 - 1,025}{4 - 7} = -\frac{175}{3}$$

Pt-slope  $y - y_1 = m(x - x_1)$   
 $\Rightarrow y - 1,200 = -\frac{175}{3}(x - 4)$

$$P(t) = -\frac{175}{3}(t - 4) + 1,200$$

Answer:  $P(t) = -\frac{175}{3}(t - 4) + 1,200$

- (b) How much was the laptop worth when it was released?

$$P(0) = -\frac{175}{3}(-4) + 1,200$$

Answer:  $\$1,433.33$

- (c) The company plans to pull the laptop off the market when its value falls below \$900. In what year would you expect this to happen?

$$P(t) = 900$$

$$\Rightarrow -\frac{175}{3}(t - 4) + 1,200 = 900$$

$$\Rightarrow -\frac{175}{3}(t - 4) = -300$$

$$\Rightarrow t - 4 = \frac{900}{175}$$

$$\Rightarrow t = \frac{900}{175} + 4 = 9.14$$

Answer:  $2013$  (will accept 2014)

6. Suppose a company manufactures computer chips. The fixed costs for this company is \$13,678 and the variable costs are \$33 per chip. The company sells each chip for \$78 a piece.

(a) Find the cost function,  $C$ , for this company, associated with selling  $q$  computer chips.

Answer:  $C(q) = 33q + 13,678$

(b) Find the revenue function,  $R$ , for this company, associated with selling  $q$  computer chips.

Answer:  $R(q) = 78q$

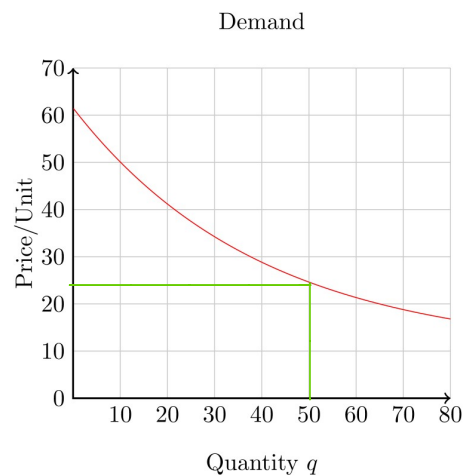
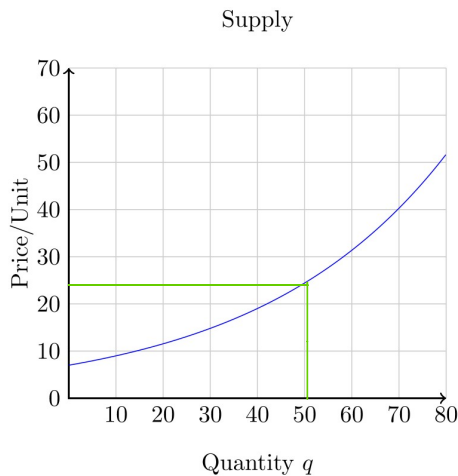
(c) Find the break-even point for this company. That is, find the number of computer chips this company must sell to break-even.

$$\begin{aligned} R(q) &= C(q) \\ \Rightarrow 78q &= 33q + 13,678 \\ \Rightarrow 45q &= 13,678 \end{aligned}$$

$$q = \frac{13,678}{45} \approx 303.956$$

Answer: 304 chips

7. Below are the supply and demand curves for a certain company. From the graphs, estimate the equilibrium point for this company.



$$\approx (50, 24) \text{ or somewhere close.}$$